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(NASA-CR-160940) AN ALGORITHM FOR THE RAPID
LOCATION OF AN EXTREME OF A FUNCTION SUBJECT
ONLY TO GEOMETRIC RESTRICTIONS (Lockheed
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AN ALGORITHM FOR THE RAPID LOCATION
OF AN EXTREMUM OF A FUNCTION SUBJECT
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ABSTRACT

If a function of a single variable is convex and symmetric in a neighborhood of an extremum, the extremum may be approximated to a precision that increases by at least a power of two per functional evaluation. This procedure may be used to drive a complex optimization procedure (such as the Davidon-Fletcher-Powell) in the kind of multivariate area estimation problem encountered in remote sensing.

Key words: Optimization, convex functions

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TECHNICAL MEMORANDUM

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160940

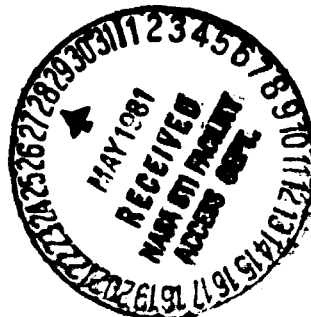
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1. INTRODUCTION

Let $f(x)$ have a minimum on an interval $[x_0, x_2]$ and assume further that f is convex upward there and symmetric around its minimum. Then we know the following fact about the minimum: (Let $x_1 = \frac{x_0 + x_2}{2}$).

Theorem: Assume without loss of generality that $f(x_0) \leq f(x_2)$. Then f assumes its minimum at a point between

$$\frac{1}{2}(x_0 + x_1) + \frac{1}{2} (x_2 - x_1) \frac{f(x_0) - f(x_1)}{f(x_2) - f(x_1)}$$

and whichever of x_0 and x_1 that has smaller functional value $f(x)$.

2. PROOF OF THEOREM

Case I: $f(x_1) \leq f(x_0) \leq f(x_2)$.

Let x^* be such that $f(x^*) = f(x_0)$ and $x_1 \leq x^* \leq x_2$. It exists by symmetry about the minimum and convexity upward. Similarly, by upward convexity $\{x^*, f(x^*)\}$ is below a segment joining the points $\{x_1, f(x_1)\}$ and $\{x_2, f(x_2)\}$ in the graph of f , so

$$f(x_0) = f(x^*) \leq f(x_1) + \frac{(x^* - x_1)}{(x_2 - x_1)} [f(x_2) - f(x_1)]$$

$$\text{So } x^* \geq x_1 + \frac{f(x_0) - f(x_1)}{f(x_2) - f(x_1)} (x_2 - x_1)$$

By symmetry of f around its minimum

$$x_{\min} = \frac{x_0 + x^*}{2}$$

So

$$x_{\min} \geq \frac{1}{2} (x_0 + x_1) + \frac{1}{2} (x_2 - x_1) \frac{f(x_0) - f(x_1)}{f(x_2) - f(x_1)}$$

$$\text{Now } x^* \leq x_2 \text{ implies } x_{\min} = \frac{x_0 + x^*}{2} \leq \frac{x_0 + x_2}{2} = x_1$$

So $x_1 \geq x_{\min}$ and we have case I.

Case II: $f(x_0) \leq f(x_1) \leq f(x_2)$

Again, let x^* be such that $f(x^*) = f(x_0)$ but $x_0 \neq x^*$. $x^* \leq x_1$ by convexity. Also by upward convexity, $\{x_1, f(x_1)\}$ is below the segment connecting $\{x^*, f(x^*)\}$ to $\{x_2, f(x_2)\}$, so

$$f(x_1) \leq f(x^*) + \frac{(x_1 - x^*)}{(x_2 - x^*)} (f(x_2) - f(x^*))$$

and this may be manipulated to

$$x^* \leq x_1 + (x_2 - x_1) \frac{f(x_0) - f(x_1)}{f(x_2) - f(x_1)}$$

Again by symmetry

$$x_{\min} = \frac{x_0 + x^*}{2}, \text{ so}$$

$$x_{\min} \leq \frac{1}{2} [x_0 + x_1] + \frac{1}{2} (x_2 - x_1) \frac{f(x_0) - f(x_1)}{f(x_2) - f(x_1)}$$

But $x_0 \leq x_{\min}$ by assumption, so we have case II.

The case $f(x_0) \leq f(x_2) < f(x_1)$ violates convexity upward, so

Q.E.D.

Corollary: The new sub-interval containing the minimum of f is at most one fourth the length of $[x_0, x_2]$.

Proof: The computed boundary in the formula is clearly from its formula nearer the other boundary than is

$$\frac{x_0 + x_1}{2} = 3/4 x_0 + 1/4 x_2.$$

Q.E.D.

3. APPLICATION

These results become an algorithm for the minimization or maximization of a function meeting or nearly meeting the requirements of symmetry and convexity. This method involves replacing $[x_0, x_2]$ by the new interval at successive iterations. Convergence is at least by powers of one fourth at a cost of two functional evaluations per iteration.